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**ABSTRACT**

Growth models are critical in the development of innovative and sustainable management strategies in aquaculture. This paper utilizes Schnute’s generalized non-linear growth model to age–length data of the Juvenile Asian seabass, *Lates calcarifer* (Bloch) under culture conditions. Steps involved in fitting the non-linear growth model were outlined. Differential growth is observed among the individuals of the same age group of the sampled data. It is observed that variance of length measurements increased over age indicating heteroscedastic nature of the data. The effect of error structure is investigated by considering additive and multiplicative errors for fitting the Schnute’s Generalized nonlinear growth model to the data. The shape of the fitted curves suggested that the practical difference is minor within the range of the observed ages. However, based on the goodness-fit-statistics, correlation structure of parameter estimates it is concluded that growth model with multiple error fitted well for the growth of juvenile Asian seabass, *Lates calcarifer* (Bloch) under culture conditions.

**KEY WORDS**

Asian seabass, culture, growth, age-length, non-linear Growth model, error structure

Mathematics Subject Classification: 62J02, 91B62

1. **INTRODUCTION**

Growth of the individual is one of the main characteristics in population dynamics. In order to understand the growth over the lifespan of the individual, several growth models have been developed. These are mathematical functions, which describe the pattern of individual growth during its lifespan. To derive an optimum harvest

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strategy for any aquaculture operation, obtaining a reliable growth model is indispensable. Such growth models are critical in the development of innovative and sustainable management strategies.

These empirical growth models can be fitted, by considering individual's size-at-age data. Here size can refer to standard length, fork length, live weight or whatever measure of the individual the biologist wish to describe. In general growth of fish follows a sigmoid growth pattern (Ricker, 1979). Such curve starts at some fixed point of time and increase their growth rate monotonically to reach an inflection point; after this the growth rate decreases to approach asymptotically to some final value.

Process producing sigmoidal or S-shaped growth models are wide spread in the literature with names such as Von-Bertalanffy growth model, Gompertz, Richards, Logistic etc. It is a difficult task to find reasonable model based on the available data. Even when a model is chosen, there are other factors to be addressed such as selecting appropriate methodology for parameter estimation, statistically good fit and proper biological interpretation. Thus in the context of precautionary approach to sustainable fishery management it is recommended to consider these model uncertainties (Butterworth et al., 1996; Mc Allister et al., 1999; Andrade and Kinas 2003). Hence to choose the more suitable models based on the available data different alternatives must be compared by some criteria.

The alternative explored in this paper is a "Generalized non-linear growth model" developed by Schnute (1981) to study the growth of juvenile Asian seabass under culture conditions. This generic growth model contains many growth models (e.g. generalized and specific Von-Bertalanffy, Gompertz, Richards, Logistic etc.) as special cases. Schnute's general formulation is attractive because it allows for a smooth transition between models of different functional forms (i.e., specific sub models) and because of the statistical stability of the resulting parameter estimates. This model is an alternative one because it is motivated by biological principlea related to growth acceleration. The parameters in this model have reasonable biological interpretation and parametric properties that allow direct use of data in selecting appropriate model systematically.

Asian seabass, *Lates calcarifer* is a catadromous centropomid perch and considered as an important candidate species for brackishwater aquaculture in India. It is a
highly potential species for farming both in culture ponds and cages as an alternative for shrimp farming, since shrimp farming is suffering from losses due to viral disease outbreaks. Its potential for farming has increased after the successful induced breeding in India (Thirunavakkarasu et al., 2001). Reports are available on biology, breeding, larval rearing techniques and culture aspects of Asian seabass (James and Marchamy, 1986, Mohammad Kasim and James, 1986, Purushan, 1990, Thirunavakkarasu et al., 2004, Kailasam et al., 2001, Kailasam et al., 2006). However, reports on growth and growth models of seabass under culture conditions are very scanty. Recently, Venugopal et al. (2003) studied fish condition and length-weight relationship of juvenile seabass under culture conditions. Since studies on growth models of seabass can predict the optimum time of harvest to achieve profitable production, this type of studies are important. Hence in the present study an attempt has been made to study the growth of juvenile Asian seabass through a generalized non-linear growth model.

2. MATERIALS AND METHODS

2.1 Data
The fry of *L. calcarifer* were collected from the Central Institute of Brackishwater Aquaculture (CIBA), Chennai, and transported to the Kakinada Centre of the Central Institute of Fisheries Education (CIFE). The fry (4000 numbers) were reared initially for 15 days with a weaning diet prepared by CIBA. After successful weaning, 600 fry each with average initial weight of 1.73 ± 0.43 g and initial length 21.20 ± 1.50 mm were randomly stocked in four ponds of 0.08 ha each. Fortnightly sampling was done by drag netting to assess the biomass and the feed quantity was adjusted accordingly. During sampling, the total length of the fish was measured from the tip of snout to the end of the caudal fin. A sample size of 809 fish were considered for the analysis.

2.2 Generalized Non-Linear Growth Model
The growth model of Schnute (1981) relates age 't' to some measure of size \( L(t) \). In the present study total length is considered as measure of size. The Schnute's generalized growth model is as follows.
Parameters $t_{1}$ and $t_{2}$ are fixed ages specified in advance with restriction $t_{1} < t_{2}$. There are four parameters to be estimated $a$, $b$, $l_{1}$, and $l_{2}$. Parameters $l_{1}$ and $l_{2}$ are size expected at ages $t_{1}$ and $t_{2}$ respectively with restriction $0 < l_{1} < l_{2}$. Various growth models known in the literature are related to specific values of parameters $a$ and $b$. If $a > 0$ and $b=1$ is specific Von-Bertalanffy growth, $a > 0$ and $b=0$ is Gompertz, $a > 0$ and $b=-1$ becomes Logistic growth and $a > 0$ and $b<0$ is Richards growth. In such cases $a$ is the intrinsic growth rate parameter usually represented by $k$ in the fishery literature (Schnute, 1981; Brey, 2001).

2.3 Error Structure of the Model

Equation 2.1 can be represented in a general form

$$L(t) = f(X_{t}, \theta)$$

(2.2)

Where $L(t) =$ value of the response variable at time $t$

$X_{t} =$ Value of the explanatory variable at time $t$

$\theta =$ Vector of unknown parameters

Equations 2.1 and 2.2 posed deterministically as if the data never deviated from the growth curve. This is unrealistic situation in natural conditions and some statistical assumptions are needed to make the model practicable. Thus, if we included an error term $e_{i}$ on the right hand side of Equations 2.1 and 2.2 then it is statistically valid.

It is to be mentioned that given a specific functional form the appropriate estimation of growth parameters also depends on the error structure assumed for the data. The error term $e_{i}$ may be either additive or multiplicative (Quinn and Deriso, 1999).

Let $(X_{i}, L_{i})$, $i=1,2,...n$ be $n$ data points where $X_{i}$ is fortnight measurement and $L_{i}$ is the length of the individual at the end of the fortnight $t_{i}$. If the variability in size is constant as a function of age, an additive structure is suitable. The resulting model can be expressed as

$$L(t) = \left[ t_{1}^{b} + \left( t_{2}^{b} - t_{1}^{b} \right) \frac{1-e^{-a(t-t_{1})}}{1-e^{-a(t-t_{2})}} \right]^{1/b} \quad a \neq 0 \text{ and } b \neq 0 \quad (2.1)$$
\[ L_i = f(X_{ui}, \theta) + \sigma_i e_i; \quad i = 1, 2 \ldots n \]  

(2.3)

Where the random variable \( e_i \) is error term and assumed to be independent and normal with zero mean and unit variance. The parameter \( \sigma_i \), which has units of size, measures the additive standard error in prediction.

If the variability of size increases as a function of age (i.e. heteroscedasticity), then it is more adequate to use a multiplicative error structure (Quinn and Deriso, 1999). Thus the model can written as

\[ L_i = f(X_{ui}, \theta) \exp(\sigma_2 e_i); \quad i = 1, 2 \ldots n \]  

(2.4)

Where \( \sigma_2 \) is a dimensionless parameter, measures the logarithmic standard prediction error.

### 2.4 Estimation of Parameters

The estimation of parameter vector \( \theta \) can be reduced to a standard linear non-linear minimization. Finding maximum likelihood estimates of \( \theta \) under additive error of Equation 2.3 to minimizing the residual sum of squares function.

\[ SS_1(\theta) = SS_1(l_1, l_2, a, b) = \sum_{i=1}^{n} \left[ L_i - f(X_{ui}, \theta) \right]^2 \]  

(2.5)

Alternatively if the multiplicative error structure in Equation 2.4 is assumed, the function is minimized is

\[ SS_2(\theta) = SS_2(l_1, l_2, a, b) = \sum_{i=1}^{n} \left[ \ln \left( \frac{L_i}{f(X_{ui}, \theta)} \right) \right]^2 \]  

(2.6)

Like linear regression, parameter estimates in non-linear case also, can be obtained by minimizing the residual sum of squares (Eqs. 2.5 and 2.6). However, because of nonlinearity, the resulting normal equations are non-linear in parameters and so cannot be solved exactly. Accordingly a number of iterative procedures have been developed to obtain approximate solution (Ratkowsky, 1983). The widely used iterative methods for nonlinear normal equations include Gauss-Newton method, Steepest-decent method, Multivariate secant or false position, and Marquardt method (Draper and Smith, 1981; Smyth, 2002).
2.5 Choice of Initial Parameters

The choice of initial starting values is very important with the iterative methods because a poor choice may result in slow convergence, convergence to a local minimum rather than the global minimum, or even divergence. Good starting values will generally result in faster convergence, and if multiple minima exist, will lead to a solution that is the global minimum rather than a local minimum.

Starting values may be obtained from previous or related studies, theoretical expectations, or a preliminary search for parameter values using some criterion. It is often desirable to try other sets of starting values after a solution has been obtained to make sure that the same solution will be found.

After choosing appropriate starting values, the search for the point of global minimum is obtained in two stages. First a simplex algorithm searches the parameter space to escape any eventual local minima. After convergence, current estimates are used as starting values in iterative procedure to get the final estimates.

The covariance matrix of estimated parameters is obtained in the usual way form the Hessian matrix of second derivative of the likelihood function.

2.6 Measures of Model Adequacy

The usual goodness-of-fit statistics, e.g. R² which used for linear models are not appropriate when the model is nonlinear (Prajneshu, 1990). Kvalseth (1985) has emphasized to test the adequacy of the fitted models apart from R² other summary statistics like RMSE and MAE should be computed. Accordingly the following goodness-of-fit statistics are computed (D’Agostiono and Stephens, 1986).

1. Root mean squared error (RMSE)

\[ \text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}[L_i - \tilde{L}_i]_i^2}{n}} \]  

2. Mean absolute error (MAE)

\[ \text{MAE} = \frac{\sum_{i=1}^{n}|L_i - \tilde{L}_i|}{n} \] 

The lower the value of these statistics the better is the model. All calculations were performed with the statistical software SPSS for Windows ver.11.0.
3. RESULTS AND DISCUSSION

Since, Asian seabass is a carnivore, many studies have been undertaken in order to achieve better growth and survival rate during larval rearing, nursery rearing and grow-out culture of seabass (Mackinnon, 1985, Van Damme et al., 1989). During rearing of seabass, cannibalism is observed and it can cause not only low survival but also differential growth rate (Kailasam, 2002). De Angelis et al., (1979) have stated that the cannibalistic nature is due to size variation among the individuals. This difference in growth among the individuals was pronounced in the sampled data.

The summary of length measurements of seabass samples collected over culture period is presented in Table 1. From the Table 1 it is observed that the range between minimum and maximum length is increased over the growth period considered. The difference between minimum and maximum length measurement is 16 mm for a 15 days old fish sample. Where as it is increased to 215 mm for 195 days old fish sample. The overall frequency distribution of length measurements in the form of histogram is presented in Figure 1 indicated that majority of fish are measured in between 80mm and 150mm.

Variances in length measurements have been calculated from the Length-at-age data and plotted in Figure 2. There is an increasing trend in the variances by age reflecting the heteroscedastic nature of the data and suggesting a multiplicative error. Schnute's generalized growth model is fitted with additive as well as multiplicative error to verify the same.

The models were fitted with several pairs of fixed ages $t_1$ and $t_2$ together with different starting values for the unknown parameters but the estimated parameters were always the same. For the results presented here, the ages are $t_1 = 15$ days, $t_2 = 195$ days and the starting values are $(a, b, l_1/l_2) = (0.125, 0.25, 20, 300)$. The results of the analysis are summarized in Table 2. All the four parameters are sensitive to the choice of additive and multiplicative error. The predicted length of Asian seabass for ages 15 and 195 days is 17.36 (5.5) and 143.35 (1.66) for the additive error model and it is 20.05(0.69) and 136.27 (2.17) for multiplicative error model respectively. The estimated values for parameter $a$ value is 0.76 (0.12) and
1.17(0.18) for additive and multiplicative error models respectively. And estimated values for parameter b are -0.25(0.48) and -1.48 (0.34) respectively. The confidence intervals of the parameters show that most of the region shared by both the models. The observed and fitted values presented in Figure 3. suggest that the practical difference is minor with in the range of observed ages. The goodness of fit statistics RMSE and MAE are lesser for multiplicative error model over additive model. It indicates that model with multiplicative error is superior to additive error model for the data considered. The strong negative correlation (>0.8) between the parameters a and b for both the models indicate high colinearity between these parameters. There is a weak negative correlation observed between I₁ and I₂ for both the models. It is slightly lesser for multiplicative error. Correlation between I₁ and a is the same for both the models. But correlation between I₁ and b is considerably low for multiplicative error when compared to additive error. This explains considerable gain in precision of estimates from additive error model to multiplicative error model. Conceptually the increase of variance in length as a function of age is an expected pattern for a fish population favouring multiplicative error. The low goodness-of-fit statistics, low correlations between the parameter estimates for multiplicative model suggest that for the present study with in the frame work of Schnute’s generalized growth model assuming multiplicative error is an acceptable choice with the parameter estimates given in Table 2. However, the possible effect of using any of these models to estimate length at age in optimum harvesting policies should be verified in the future.

REFERENCES


Table 1: Summary of Length measurements collected at fortnightly intervals.

<table>
<thead>
<tr>
<th>Age (days)</th>
<th>Sample Size</th>
<th>Minimum Length (mm)</th>
<th>Maximum Length (mm)</th>
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<tr>
<td>15</td>
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<td>16</td>
<td>32</td>
</tr>
<tr>
<td>30</td>
<td>20</td>
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<td>101</td>
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<tr>
<td>60</td>
<td>19</td>
<td>81</td>
<td>129</td>
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<td>75</td>
<td>118</td>
<td>90</td>
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<td>180</td>
<td>98</td>
<td>88</td>
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<tr>
<td>195</td>
<td>114</td>
<td>85</td>
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Table 2: Schnute’ Generalized nonlinear Growth model parameter estimates, Goodness-of-fit statistics and Correlations for the Asian seabass, *Lates calcarifer* (Bloch) under culture conditions.

<table>
<thead>
<tr>
<th>Additive Error Model</th>
<th>Multiplicative Error model</th>
<th>95% CI</th>
<th>95% CI</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Lower</td>
<td>Upper</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20.05</td>
<td></td>
</tr>
<tr>
<td>L₁</td>
<td>17.36</td>
<td>5.50</td>
<td>6.46</td>
</tr>
<tr>
<td>L₂</td>
<td>143.35</td>
<td>1.66</td>
<td>140.09</td>
</tr>
<tr>
<td>a</td>
<td>0.71</td>
<td>0.12</td>
<td>0.47</td>
</tr>
<tr>
<td>b</td>
<td>-0.25</td>
<td>0.48</td>
<td>-1.19</td>
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Goodness-of Fit Statistics

<table>
<thead>
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<th>RMSE</th>
<th>MAE</th>
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<tr>
<td>32.46</td>
<td>4.91</td>
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<td>30.84</td>
<td>3.90</td>
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Correlations

<table>
<thead>
<tr>
<th>L₁, L₂</th>
<th>L₁, a</th>
<th>L₁, b</th>
<th>L₂, a</th>
<th>L₂, b</th>
<th>a, b</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.06</td>
<td>-0.41</td>
<td>-0.76</td>
<td>-0.45</td>
<td>0.28</td>
<td>-0.88</td>
</tr>
</tbody>
</table>
Figure 1: Variances of length measurements (mm$^2$) over the age (days)

Figure 2: Histogram showing the frequency distribution of overall length measurements.
Figure 3: Schnute's models assuming additive and multiplicative error fitted to data from Asian seabass, *Lates calcarifer* (Bloch) under culture conditions.